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On the Detection of Ultrawideband Radar Signals

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The detection of ultrawideband (UWB) radar signals is considered. Conventional radar theory is examined in the context of ultrawideband signals and extended when necessary. Point target detectors are found for the cases when the additive interference is white Gaussian noise and correlated Gaussian noise or clutter. A conventional moving target indicator (MTI) design is generalized for UWB radar. The effects of bandwidth on the resolution of clutter and target are examined. Optimal and conventional processing are compared for single echo detection. An optimal detector is found for a distributed target with a known target profile. Additionally, optimal detectors are found under two models where only second-order statistics of the target profile are assumed known. Future work areas are suggested.

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CONTENTS

ON THE DETECTION OF ULTRAWIDEBAND RADAR SIGNALS

1. INTRODUCTION

An ultrawideband (UWB) radar is a radar that has an instantaneous bandwidth which is relatively large. An impulse radar, which is one type of UWB radar, is a radar that has a time-bandwidth product nearly equal to 1. An introduction to impulse radar can be found in Ref. 1.

The primary use of UWB radar is in applications in which conventional radars do not perform satisfactorily. Fowler et al. [2] concluded, however, that impulse radars

- show no evidence of phenomenological advantages,
- have no unique advantages in terms of anti-stealth, and
- possess no special low probability of intercept (LPI) advantages.

But Fowler et al. also concluded that UWB radars may prove to be more effective over conventional radars in certain short-range applications which require high range resolution (HRR) such as terrain profiling. Reference 3 lists target identification, detection of specific air-turbulence patterns, and detection of low radar cross section targets as other potential HRR applications.

An obstacle to implementing a true wideband radar is the tremendous amount of data which needs to be processed in real time. Another problem is in generating the energy, within a period typically on the order of picoseconds, necessary to detect or discriminate a target.

This report examines general signal processing aspects of UWB radar from a detection standpoint. In Section 2, we model the process and derive a detector structure for a point target in white Gaussian noise. Section 3 derives a new generalized moving target indicator (MTI) detector design for cases of correlated Gaussian noise or clutter, and extends the detector to a generalized Doppler filter bank. Additionally, the associated effects of bandwidth on performance in the presence of clutter are examined. Section 4 considers range spread targets and derives three fundamental detector structures. Section 5 discusses future work areas and summarizes the report.

It should be mentioned that much of the work is based on extending, when necessary, conventional radar processing theory taken from Van Trees [4,5].

2. POINT TARGET DETECTION

We will now derive the optimal detection statistic for an UWB signal assuming a point target. We assume the (single pulse) transmitted waveform is given by

$$\sqrt{2}\, Re\left[\sqrt{E_t}\, f(t)\ \exp(j\omega_c t)\right],$$

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where E_t is a constant representing the transmitted energy of the signal and $f(t)$ is the complex envelope of the transmitted signal ω_c, the carrier frequency. It is assumed that

$$\int |f(t)|^2 dt = 1.$$

We assume that the propagation and reflection processes are linear and frequency-independent, and that the target is at a distance R_o at $t = 0$ and is moving with constant radial velocity v. Under these circumstances, it is straightforward to show [4, p. 241] that the received signal is

$$\sqrt{2} Re\left[\sqrt{E_t}\, \tilde{b} f(t - \tau(t)) \exp(j\omega_c(t - \tau(t)))\right],$$

where

$$\tau(t) = \frac{2R_o/c}{1 - v/c} - \frac{2vt/c}{1 - v/c}$$

is the round-trip time delay, $\tilde{b}$ represents an unknown complex amplitude, which is often assumed to have a zero mean Gaussian distribution. We can reduce $\exp(j\omega_c(t - \tau(t)))$ by absorbing the first term of $\tau(t)$ into $\tilde{b}$ and defining $\omega_d = \dfrac{2v\omega_c/c}{1 - v/c}$ as

$$\exp(j\omega_c(t - \tau(t)) \propto \exp(j(\omega_c + \omega_d)t).$$

To simplify $f(t - \tau(t))$, the approximation $\tau(t) \approx 2R_o/c - 2vt/c$ is made for conventional radars. However, the actual time the signal return begins (which we will refer to as t_b) assuming f begins at $t = 0$, is when $\tau(t_b) = t_b$, which simplifies to

$$t_b = \frac{2R_o/c}{1 + v/c},$$

whereas the latter approximation yields

$$t_b \approx \frac{2R_o/c}{1 + 2v/c}.$$

Now suppose we are trying to detect a high-speed target with $R_o = 20{,}000$ m and $v = 600$ m/s. The approximation yields $\tau_b \approx 1.333328E - 4$ whereas the actual time is $\tau_b \approx 1.333301E - 4$, a difference of $2.7-10$. Such a difference may seem insignificant until it is compared with the extent of a typical pulse. For example, assuming a bandwidth of 5 GHz, the pulse width is on the order of a few hundred ps, or of the same order as the time difference between the approximated value and the actual value. Thus, we could be in range error by as much as one range cell width when using this approximation. We

can now accurately approximate $f(t - \tau(t))$ by $f(t - \tau)$ where $\tau = \dfrac{2R_o/c}{1 + v/c}$. Note that we have removed the dependence of $\tau(t)$ on t by replacing $\tau(t)$ by the epoch t_b of the received pulse. This is valid as long as the compression or stretching of the time scale caused by the dependence on time is not significant. It is shown in Ref. 4 (p. 241) that this is the case whenever the time bandwidth product of the received signal is $\ll c/(2v)$; this will be assumed for the UWB signals considered here.

We assume that one of two hypotheses is received. The hypotheses are signals present with additive noise $n(t)$ under H_1 and only additive noise under H_0. Under hypothesis H_1 we receive

$$r(t) = \sqrt{2}Re\left[\sqrt{E_t}\,\tilde{b}f(t - \tau)\exp(jt(\omega_c + \omega_d)) + n(t)\exp(j\omega_c t)\right], \tag{1}$$

while under H_0

$$r(t) = \sqrt{2}Re\left[n(t)\exp(j\omega_c t)\right] \tag{2}$$

is received. ω_d is an added Doppler frequency shift. We now make the additional assumption that the signal is band-limited. Under these circumstances, there exist the transformations

$$r_c(t) = (\sqrt{2}r(t)\cos\omega_c t)_{LP}$$

$$r_s(t) = (\sqrt{2}r(t)\sin\omega_c t)_{LP}$$

$$r(t) = \sqrt{2}r_c(t)\cos\omega_c t + \sqrt{2}r_s(t)\sin\omega_c t$$

where $(\cdot)_{LP}$ represents a low-pass operation. Note these operations are invertible, hence by the factorization theorem [5] it can be shown that the problem is equivalent to discriminating between

$$\tilde{r}(t) = \sqrt{E_t}\,\tilde{b}f(t - \tau)\exp(j\omega_d t) + n(t) \text{ under } H_1 \text{ and}$$

$$\tilde{r}(t) = n(t) \text{ under } H_o. \tag{3}$$

This problem has the solution [4, p. 245]

$$\left|\int_o^{T_P} \tilde{r}(t)f^*(t - \tau)\exp(-j\omega_d t)dt\right|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \gamma\,,$$

where T_P is the pulse repetition interval and γ is a threshold associated with the probability of false alarm. The test is uniformly most powerful for the case where $\tilde{b} = \zeta\exp(j\phi)$, ζ is unknown and ϕ is a uniformly distributed random variable on $[0,2\pi]$. To prove this, examine Ref. 5 (p. 339). The test is optimal in the case where $\tilde{b}$ is assumed Gaussian distributed with zero mean, which is the case when ζ

is Rayleigh distributed and ϕ is uniformly distributed on [0, 2π][7]. For the case of n pulses, this becomes

$$\left| \int_0^{nT_p} r(t) \sum_{i=0}^{n-1} f^*(t - \tau_i) \exp(-j\omega_d t) dt \right|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

or

$$\left| \sum_{i=0}^{n-1} \int_0^{nT_p} r(t) f^*(t - \tau_i) \exp(-j\omega_d t) dt \right|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \gamma, \tag{4}$$

where $\tau_i = \dfrac{2(R_0 - iv T_P)/c}{1 + v/c} + iT_P \quad i = 0, \ldots, n-1$. This detector assumes knowledge of $\omega_d = \dfrac{2v\omega_c/c}{1 + v/c}$ which, in practice, is typically not known. An implementable detector structure is developed in Section 3 that does not assume knowledge of ω_d and is extended from a white noise assumption to correlated noise or clutter.

3. DETECTION IN CLUTTER

In the derivation of Eq. (4), the target is not assumed to lie in the same range cell from pulse to pulse. The associated delays τ_i are dependent upon the target velocity in contrast to a conventional radar where $\tau_i = iT_P$ is nearly velocity independent. In the presence of clutter or correlated noise, this difference has important consequences. In the conventional case, the clutter within the target range cell can be cancelled using only data from returns of the target at the same range cell. Hence, the same data can be used to both cancel clutter and integrate the target energy from successive pulses. In the UWB case, however, the target may move through range cells between successive pulses so that the range cells that contain the target may not be sufficient to cancel the clutter. That is, the degree of clutter correlation between successive target returns that lie in different range cells may be much less than in the case of conventional radars. In general, we would require additional data that do not contain the target to fully cancel the clutter and obtain reasonable detection performance.

We refer again to the hypothesis testing problem of Eqs. (1) and (2) where we model the clutter $n(t)$ by a correlated zero-mean nonwhite Gaussian noise process. The optimal detector is found by using standard techniques

$$\left| \int_0^{nT_p} r(t) g^*(t) \exp(-j\omega_d t) dt \right|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \gamma, \tag{5}$$

where $g(t)$ is the pseudosignal formed by solving

$$\sum_{i=0}^{n-1} f(t - \tau_i) = \int_0^{nT_p} K_n(t,u) g(u) du$$

and $K_n(t,u) = E(n(t)n^*(u))$. The pseudosignal would take into account correlation in range as well as time correlation from pulse to pul.e.

Generally, the Doppler ω_d is unknown. A bank of filters of the form of Eqs. (4) or (5) may be used such that each filter is matched to a different Doppler shift. When the maximum of these filters is compared with a threshold, the resulting test is often referred to as a generalized likelihood ratio test (although an infinite number of filters is technically required). In this case, we would require a number of integrators to perform the computation of either Eqs. (4) or (5).

Let us denote the output of a filter matched to $f(t)$ by

$$u(t) = \int r(\tau) f^*(\tau - t) d\tau. \tag{6}$$

Suppose that nearly all the energy of $f(t)$ is concentrated within the time interval $[0, T_f]$. If $\omega_d \tau$ varies relatively little during this interval, we can make the approximation [8]

$$u(t) \approx \exp(j\omega_d t) \int r(\tau) f^*(\tau - t) \exp(-j\omega_d \tau) d\tau.$$

For the case of white Gaussian noise, the detector structure of Eq. (4) then reduces to the optimal solution

$$\left| \sum_{i=0}^{n-1} \exp(-j\omega_d \tau_i) u(\tau_i) \right|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \gamma. \tag{7}$$

Although Eq. (6) is a sufficient statistic for the detection problem in white Gaussian noise, it is not in general sufficient for arbitrary correlated noise. However, in the case of a conventional radar $(\tau_i = iT_p)$, a complex weighting a_i is typically substituted for the values $\exp(-j\omega_d \tau_i)$ in Eq. (7). The

use of an appropriate weighting is well known to eliminate the effects of most clutter. The resulting detector is then

$$\left| \sum_{i=0}^{n-1} a_i u(\tau_i) \right|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \gamma . \tag{8}$$

This detector for appropriate a_i is referred to as an MTI. A bank of such filters is a Doppler filter bank.

In the case of UWB operating in clutter, the detector structure of Eq. (8) may not be sufficient to cancel the clutter. This is due to the loss of correlation between the $u(\tau_i)$ which may occur if the target moves between range cells on successive pulse repetition intervals. A generalization of Eq. (8) is thus

$$\left| \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} a_{ij} u(\tau_i + T_P(j - i)) \right| \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{9}$$

where n is the number of pulses. We define a detector of the form described in Eq. (9) to be a generalized MTI (GMTI), and the n by n matrix A of elements a_{ij} to be the GMTI coefficients. We can extend the GMTI to a generalized Doppler filter bank as follows. Let $a_{ij}^{(\omega_d)}$ be a set of Doppler coefficients matched to a target with associated Doppler ω_d. We can compare each filter with a threshold or declare a detection whenever

$$\max_{\omega_d} \left| \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} a_{ij}^{(\omega_d)} u(\tau_i + T_P(j - i)) \right| \underset{H_0}{\overset{H_1}{\gtrless}} \gamma .$$

Performance in the white Gaussian noise case is well known. We can relate the probability of detection P_d with the probability of false alarm P_f [4, p. 246]

$$P_f = P_d^{1+\Delta} \tag{10}$$

where $\Delta = \bar{E}/N_0$, $\bar{E}$ is the expected value of the received signal energy and N_0 is the (complex) noise spectral density. In the case of correlated Gaussian noise, Eq. (10) holds although $\Delta = \bar{E}\int f(t) g^*(t) dt$. In either case, performance is limited by the spectral density of the thermal noise and the received signal energy. This is also the case for conventional radar.

3.1 Computation of GMTI Coefficients

In this section we show how to compute the GMTI coefficients of Eq. (9). Consider the vector **x** where $x_o = u(\tau_0)$, $x_1 = u(\tau_0+T_P)$, ..., $x_{n-1} = u(\tau_0+(n-1)T_P)$, $x_n = u(\tau_1-T_P)$, $x_{n+1} = u(\tau_0)$, $u(\tau_0)$, $x_{2n-1} = u(\tau 1 + (n-2)T_P)$ continuing through x_{n^2-1}. In general, $x_{in+j} = u(\tau_i + T_P(j-i))$ where $0 \le i \le n-1$ and $0 \le j \le n-1$ (see Fig. 1). We see that the entries $x_{in}, x_{in+1}, \ldots, x_{in+n-1}$ will be correlated because of the temporal correlation of clutter, while $x_j, x_{n+j}, \ldots, x_{(n-1)n+j}$ will be correlated because of the spatial correlation of clutter. Other cases represent combinations of both time and spatial correlation. Assuming stationarity, we can represent the covariance matrix R of x by

$$R = E(\mathbf{x}\mathbf{x}^{\mathrm{H}}) = \begin{bmatrix} R_t & R_{s_1^H} & \cdots & R_{s_{n-2}} & R_{s_{n-1}} \\ R_{s_1} & R_t & \cdots & R_{s_{n-1}} & R_{s_{n-2}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ R_{s_{n-2}^H} & R_{s_{n-1}^H} & \cdots & R_t & R_{s_1} \\ R_{s_{n-1}^H} & R_{s_{n-2}^H} & \cdots & R_{s_1} & R_t \end{bmatrix}$$

where R_t represents the covariance matrix associated with any row of Fig. 1 and R_{s_i} is the cross covariance matrix associated with any two rows of Fig. 1 that are separated by i rows. $\mathbf{x}^{\mathrm{H}}$ denotes the Hermitian transpose of **x**. Hence, R_t is a clutter covariance matrix that may have relatively large values, while the matrices R_{s_i} will typically diminish with increasing i as the clutter decorrelates spatially.

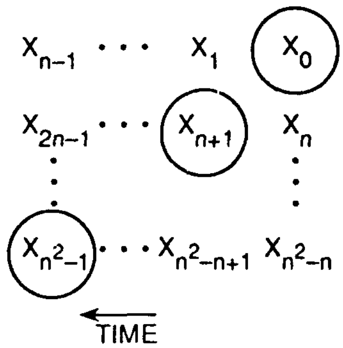

Fig. 1 — Matched filter output array for a target with range decreasing in time. Target occupancy is x_0, x_{n+1} ..., x_{n^2-1}.

Assuming Gaussian noise $n(t)$, the vector **x** is easily shown to be Gaussian. We can formulate the new detection problem as

$$y = \mathbf{x} + \mathbf{s}\exp(j\phi) \text{ under } \mathrm{H}_1 \text{ and}$$
$$y = \mathbf{x} \text{ under } \mathrm{H}_0,$$

where s is the expected value of x under H_1 and ϕ is a random uniformly distributed phase. It is known [9] that the optimal solution is

$$\left| \sum_{i=0}^{n^2-1} c_i x_i \right| \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{11}$$

where $\mathbf{c} = \mathbf{R}^{-1}\mathbf{s}^*$. To illustrate this, we assume for simplicity that $n = 2$, $\mathbf{s} = (1,0,0,1)^H$ and

$$R = \begin{bmatrix} 1 & \rho & 0 & 0 \\ \rho & 1 & 0 & 0 \\ 0 & 0 & 1 & \rho \\ 0 & 0 & \rho & 1 \end{bmatrix}.$$

The optimal coefficient vector is then given by $(1,-\rho,-\rho,1)$. As ρ approaches one, the optimal solution becomes

$$\left| x_o - x_1 - x_2 + x_3 \right| \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

which essentially removes the clutter and integrates the signal. Note that the signal only lies in two components. A similar result could easily be derived to allow implementation with a transversal filter. Hence, in cases such as this, Eq. (8) would not be adequate. This situation is extreme in that the target returns are separated by at least a range cell between pulse repetition intervals. In cases where this does not occur, the complex representation of Eq. (3) is very useful, as the optimal detector of Eq. (11) will incorporate the phase change of the signal relative to the carrier frequency, allowing relatively slow targets to be detected.

3.2 Spatial Correlation

Increasing bandwidth or resolution may have the effect of spatially correlating range cells compared with a narrowband radar. The effect that spatial correlation might have on the signal-to-noise ratio can be determined via measurements made to estimate the covariance matrix R. Let R_t be the covariance matrix where entries due to spatial correlation are zero. It is known [9] that the optimal improvement in signal-to-noise ratio is given by

$$\frac{1}{\gamma}\left|\mathbf{s}^T \mathbf{R}^{-1} \mathbf{s}^*\right|, \tag{12}$$

where s is any signal vector corresponding to Eq. (11) and γ is the received signal-to-noise ratio. We simply mention here that a measure of relative performance is given by

$$\delta = \frac{\left|\mathbf{s}^T \mathbf{R}^{-1} \mathbf{s}^*\right|}{\left|\mathbf{s}^T \mathbf{R}_t^{-1} \mathbf{s}^*\right|}.$$

3.3 Clutter Resolution

Increasing the resolution will have an impact on a radar's ability to resolve a limited number of clutter scatterers. The problem will be examined by considering the processing of a single transmitted pulse without Doppler processing. The environment is assumed to consist of a target between two clutter scatterers. The detection performance is investigated as a function of both the bandwidth and the distance a target is between the two clutter scatterers. The general resolution problem was studied in detail by Van Trees [4, pp. 323-335], although we are interested in the effects of bandwidth on clutter resolution.

We fix the distance between the two clutter scatterers at 1 m for the examples to follow simply to illustrate the effects of bandwidth on clutter resolution by conventional and optimal processing. We assume that a point target is located at a distance d from clutter scatterer number 1 as indicated in Fig. 2. Let $f(t)$ be the received signal and for simplicity zero target Doppler is assumed. The received signals from scatterers 1 and 2 are, respectively, $\tilde{b}_1 f(t - \tau_1) \exp(j\omega_1 t)$ and $\tilde{b}_2 f(t - \tau_2) \exp(j\omega_2 t)$, where $\tau_1 = -2d/c$, and $\tau_2 = 2(1 - d)/c$. Hence we formulate the hypothesis

$$r(t) = \sqrt{E_t}\, \tilde{b}_0 f(t) + \sqrt{E_t} \sum_{i=1}^{2} \tilde{b}_i f(t - \tau_i) \exp(j\omega_i t) + n(t) \quad \text{under } H_1 \text{ and}$$

$$r(t) = \sqrt{E_t} \sum_{i=0}^{2} \tilde{b}_i f(t - \tau_i) \exp(j\omega_i t) + n(t) \text{ under } H_o.$$

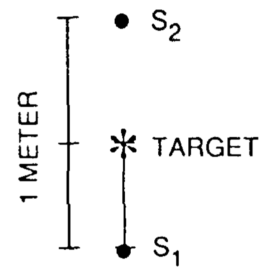

Fig. 2 — Depiction of clutter scenario. S_1 and S_2 are clutter scatterers separated by 1 meter. Target is at d meters from S_1.

The $\tilde{b}_i$ are assumed to be zero mean complex Gaussian random variables such that $E(\tilde{b}_i \tilde{b}_j^*) = 2\sigma_i^2 \delta_{ij}$ where $\delta_{ij} = 1$ for $i = j$ and 0 otherwise. $n(t)$ is zero mean white Gaussian noise with covariance function $E(n(t)n^*(u)) = N_0 \delta(t - u)$. We first assume the detector is of the conventional type, and later assume the detector is optimal. In either case, the performance of this detector can be derived as [4, p. 251]

$$P_{fa} = P_d^{1+\Delta}$$

where the performance exponent $\Delta = \Delta_c$ for the conventional receiver and $\Delta = \Delta_o$ for the optimal receiver. The conventional detector is given by

$$\left| \int r(t) f^*(t) dt \right| \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

and the performance coefficient [4, p. 327] is given by

$$\Delta_c = \frac{\dfrac{\overline{E_0}}{N_0}}{1 + \dfrac{\overline{E_1}}{N_0} \theta(\tau_1, \omega_1) + \dfrac{\overline{E_2}}{N_0} \theta(\tau_2, \omega_2)}, \tag{13}$$

where $\overline{E_i} = 2 E_t \sigma_i^2$, and $\theta(\tau_i, \omega_i)$ is the ambiguity function of $f(t)$,

$$\theta(\tau, \omega) = \left| \int f(t - \frac{\tau}{2}) f^*(t + \frac{\tau}{2}) \exp(j\omega t) dt \right|^2 .$$

The conventional receiver is not optimal due to the inclusion of the clutter scatterers. It can be shown [4, p. 330] that the optimal receiver is

$$\left| \int r(t) g^*(t) dt \right| \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

where

$$g(t) = g_o f(t) + \sum_{i=1}^{2} g_i f(t - \tau_i) \exp(j\omega_i t)$$

and the coefficients g_i are computed as follows. Define:

$$f_I = \begin{bmatrix} f(t - \tau_1) \exp(j\omega_1 t) \\ f(t - \tau_2) \exp(j\omega_2 t) \end{bmatrix}$$

$$\Lambda = \begin{bmatrix} \overline{E_1} & 0 \\ 0 & \overline{E_2} \end{bmatrix}$$

$$\rho = \int f_I(t) f_I(t)^H dt$$

$$\rho_d = \int f_I^*(t) f(t) dt.$$

Then $g_o = \frac{1}{N_o}$ and

$$g \equiv \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = -\frac{1}{N_o^2}\left[I + \frac{1}{N_o}\Lambda\rho^*\right]^{-1}\Lambda\rho_d .$$

Additionally, the optimal exponent Δ_o can be derived as

$$\Delta_o = \frac{\overline{E_o}}{N_o}\left[1 - \frac{1}{N_o}\rho_d^H\left(I + \frac{1}{N_o}\Lambda\rho^*\right)^{-1}\Lambda\rho_d.\right. \tag{14}$$

We assume $\omega_i = 0$ and $\overline{E_i} = C/N$ for $i = 1,2$, and define $R_c = \frac{\Delta_c}{\overline{E_o}/N_o}$ and $R_o = \frac{\Delta_o}{\overline{E_o}/N_o}$. R_c and R_o represent normalized signal-to-noise ratios after conventional and optimal filtering, respectively. In this case, Eq. (13) can be rewritten as

$$R_c = \frac{1}{1 + \frac{\overline{E_1}}{N_o}\theta(\tau_1,0) + \frac{\overline{E_2}}{N_o}\theta(\tau_2,0)}, \tag{15}$$

and Eq. (14) as

$$R_o = 1 + \frac{1}{\left[1 + \frac{1}{C/N}\right]^2 - \theta\left[\tau_1 - \tau_2,0\right]}\left[\left[1 + \frac{1}{C/N}\right]\left(\theta\left(\tau_1,0\right) + \theta\left(\tau_2,0\right)\right.\right.$$

$$\left. -2\sqrt{\theta\left(\tau_1 - \tau_2,0\right)\theta\left(\tau_1,0\right)\theta\left(\tau_2,0\right)}\right)\Big], \tag{16}$$

where τ_1 and τ_2 are parameterized by d as in Fig. 2. Hence, the performance of both the conventional and optimal receivers depends on the ambiguity function of $f(t)$. For example, let $f(t)$ be a Gaussian pulse

$$f(t) = \left(\frac{1}{\pi T^2}\right)^{\frac{1}{4}}\exp\left(-\frac{t^2}{2T^2}\right).$$

It is known that the ambiguity function is [4, p. 283]

$$\theta(\tau,\omega) = \exp\left[-\frac{1}{2}\left(\frac{\tau^2}{T^2} + T^2\omega^2\right)\right],$$

where T is a constant proportional to the useful duration of the pulse. It is easily shown that the power spectrum of $f(t)$ is proportional to $e^{-T^2\omega^2}$ and, hence, the 3 dB bandwidth $\omega_{3dB} = \frac{\sqrt{\ln 2}}{T}$ radians per second.

Assuming a Gaussian pulse, we plot in Figs. 3(a)-3(d) R_c vs d, and in Figs. 4(a)-4(d), R_o vs d, for various C/N ratios and ω_{3dB} bandwidths. It is seen that at 100 MHz, detection performance is limited by the clutter residue, while at higher frequencies, much of the region is free of the clutter from a detection standpoint. Also note in cases such as ω_{3dB} = 1 GHz the optimal solution substantially improves subclutter visibility of the target echo over the conventional processor.

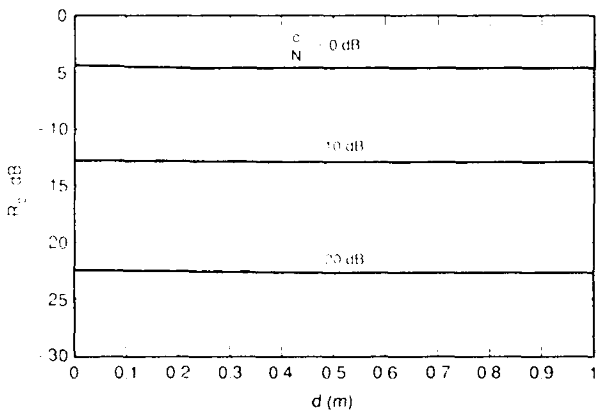

Fig. 3(a) — R_c vs d, frequency = 100 MHz, conventional processor

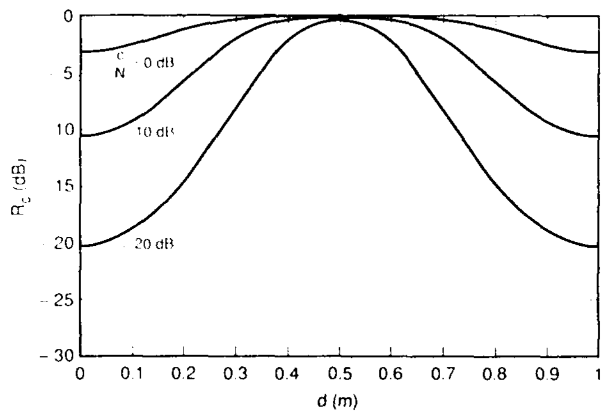

Fig. 3(b) — R_c vs d, frequency = 1 GHz, conventional processor

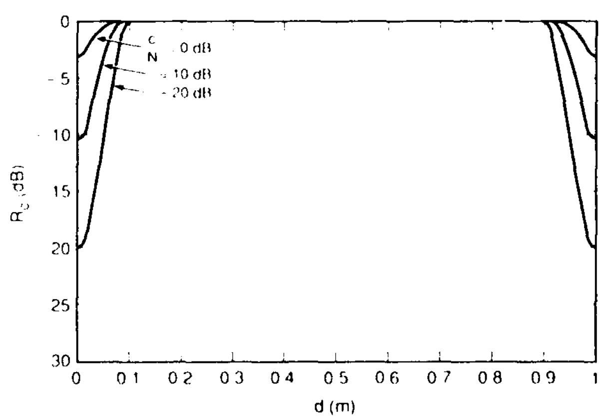

Fig. 3(c) — R_c vs d, frequency = 5 GHz, conventional processor

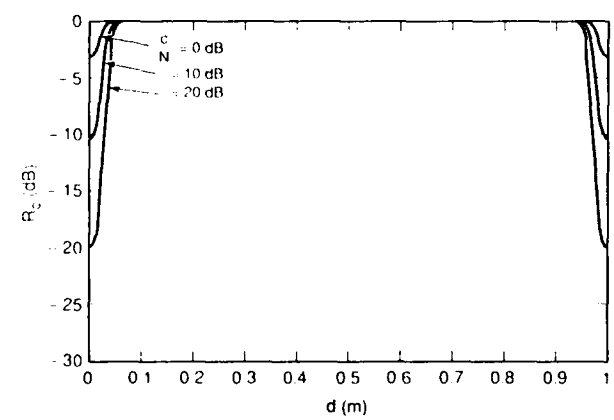

Fig. 3(d) — R_c vs d, frequency = 10 GHz, conventional processor

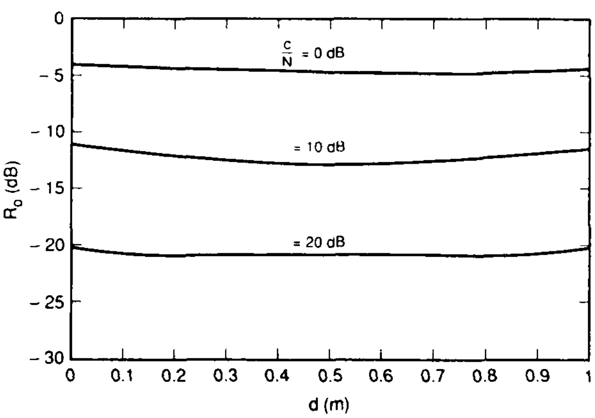

Fig. 4(a) — R_o vs d, frequency = 100 MHz, optimal processor

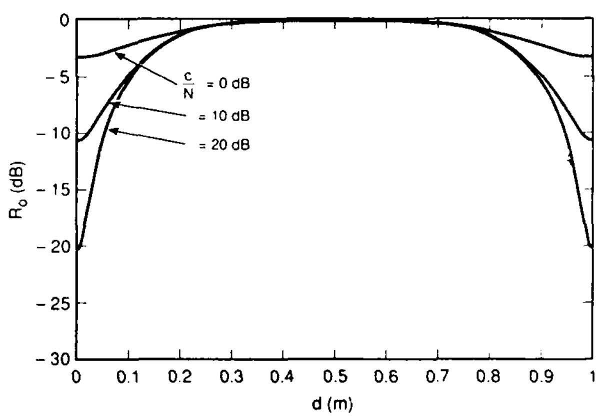

Fig. 4(b) — R_o vs d, frequency = 1 GHz, optimal processor

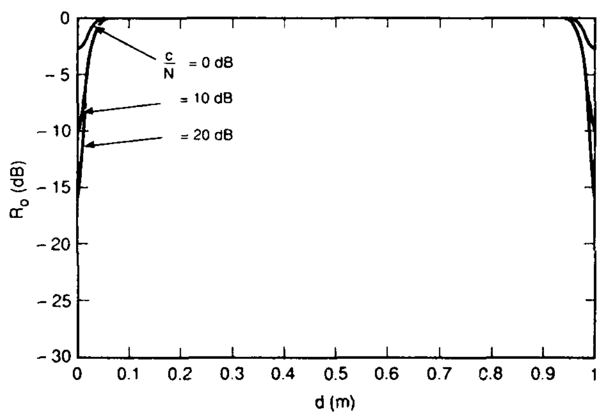

Fig. 4(c) — R_o vs d, frequency = 5 GHz, optimal processor

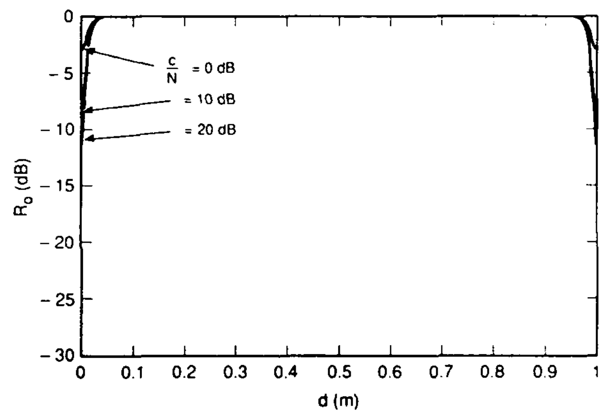

Fig. 4(d) — R_o vs d, frequency = 10 GHz, optimal processor

Implementation of the optimal processor requires knowledge of the positions of the clutter scatterers and their relative strengths. Such processing, however, may be achieved adaptively by estimating the clutter strengths and positions assuming a weak target signal. Such a processor may be useful to reduce the effects of clutter when postprocessing is not sufficient to fully cancel clutter.

4. RANGE SPREAD TARGETS

Due to the fine resolution of HRR radars, target returns will typically occupy more than one range cell. We consider three cases. In the first case, the target amplitude profile is assumed known up to a proportionality constant. An optimal detector is derived which is an extension of Eq. (7). In the remaining two cases, the target profile is not known exactly, although certain second order statistics are assumed known regarding the target. For the second case, we assume that the target scatterers individually resolve and model the return as a sum of resolved point scatterers with statistically independent Gaussian amplitudes. For the final case, the target is modeled by a weighted integral of the transmitted signal where the weighting function is assumed zero mean Gaussian with known covariance

function. The returns from different ranges are assumed statistically independent. An optimal detector is found via a low energy coherence (LEC) assumption [4, p. 431]. The LEC case requires that the eigenvalues λ_i in the Karhunen-Loève expansion of the signal $s(t)$ are small compared to the spectral density of the white noise level. Since the signal energy is spread across a large bandwidth in UWB radar, this is a reasonable assumption.

For simplicity, we will assume the target is embedded in white Gaussian noise and consider the hypothesis test in Eq. (3). Suppose $f(t)$ can be written as

$$f(t) = \int b(\lambda)s(t - \lambda)d\lambda,$$

where $b(\lambda)$ is a known complex function proportional to the amplitudes of the target scatterers and $s(t)$ is the transmitted signal. Let us denote the output of a filter matched to $s(t)$ by

$$u_s(t) = \int r(\tau)s^*(\tau - t)d\tau. \tag{17}$$

The optimal receiver of Eq. (7) is given by

$$\left| \sum_{i=0}^{n-1} \exp(-j\omega_d\tau_i)u(\tau_i) \right| \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

$$= \left| \sum_{i=0}^{n-1} \exp(-j\omega_d\tau_i) \int r(\tau)f^*(\tau - \tau_i)d\tau \right| \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

$$= \left| \sum_{i=0}^{n-1} \exp(-j\omega_d\tau_i) \int r(\tau) \left[\int b^*(\lambda)s^*(\tau - \tau_i - \lambda)d\lambda \right] d\tau \right| \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

$$= \left| \sum_{i=0}^{n-1} \exp(-j\omega_d\tau_i) \int b^*(\lambda)u_s(\tau_i + \lambda)d\lambda \right| \underset{H_0}{\overset{H_1}{\gtrless}} \gamma. \tag{18}$$

Note that if the individual target scatterers are resolved, the term $\int b^*(\lambda)u_s(\tau_i + \lambda)d\lambda$ essentially is a coherent addition of the individual target scatterers energies. If the scatterers do not resolve, the amplitudes will typically be such that some cancellation occurs between the scatterers resulting in a loss of energy. This effect was noted by Farina [3] who showed that for a moderate number of scatterers,

significant improvements in the detection performance are possible by using an UWB over a low-resolution radar.

In the second case, we will assume the target scatterers are resolved and that their amplitudes are independent Gaussian random variables. That is,

$$f(t) = \sum_{i=0}^{K-1} \tilde{b}_i s(t - \lambda_i)$$

where the $\tilde{b}_i$ are zero mean Gaussian with $E(\tilde{b}_i \tilde{b}_j^*) = 2\sigma_i^2 \delta_{ij}$. If the λ_i are separated enough so that $s(t - \lambda_i)$ can be assumed orthonormal, the signal covariance matrix is separable, and the optimal solution is [6, p. 352]

$$\sum_{i=0}^{K-1} \frac{2\sigma_i^2}{2\sigma_i^2 + N_o} \left| \int \tilde{r}(t) s(t) dt \right| \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

For n multiple pulses, $\sum_{i=0}^{n-1} f(t - \tau_i)$, we will assume that the scatterers maintain the same geometric phase relation from pulse to pulse resulting in the detection statistic

$$\sum_{i=0}^{K-1} \frac{2\sigma_i^2}{2\sigma_i^2 + N_o} \left| \sum_{j=0}^{n-1} \exp(-j\omega_d \tau_i) \int \tilde{r}(t) s(t - \lambda_i - \tau_j) dt \right|^2 \underset{H_0}{\overset{H_1}{\gtrless}} \gamma . \tag{19}$$

Again we see that the individual scatterer energy is added to form the detection statistic.

Generally the function $b(\lambda)$ is highly target dependent, or may not be known. Suppose $b(\lambda)$ is modeled as a sample function from a zero mean complex Gaussian process $\tilde{b}(\lambda)$ such that

$$f(t) = \int \tilde{b}(\lambda) s(t - \lambda) d\lambda$$

and furthermore assume the covariance function of $\tilde{b}(\lambda)$ is of the form

$$K_{\tilde{b}}(\lambda_1, \lambda_2) = \delta(\lambda_1 - \lambda_2) S(\lambda_1)$$

where $S(\lambda) = E(|\tilde{b}(\lambda)\tilde{b}^*(\lambda)|^2$, and $\delta(\cdot)$ represents the impulse function. The optimal solution in general is difficult to implement because there can be an infinite number of eigenvalues/eigenfunctions in the Karhunen-Loève expansion of $K_{\tilde{b}}(\lambda_1, \lambda_2)$. However if we make the LEC assumption that the eigenvalues in a Karhunen-Loève expansion are much less than the spectral density of the white Gaussian noise, approximations can be used which simplify the detector structure. For the UWB case, this is typically the case for weak targets due the wide bandwidth of the signal. For strong targets, the detector

structure, although suboptimum, may still perform sufficiently well, since the strength of the target may suffice to overcome the deficiencies of the suboptimum detector. For a single pulse, the optimal detector is [4, p. 431]

$$\int S(\lambda)|u_s(\lambda)|^2 d\lambda \underset{H_0}{\overset{H_1}{\gtrless}} \gamma,$$

which is easily extended to n multiple pulses $\sum_{i=0}^{n-1} f(t - \tau_i)$,

$$\int S(\lambda)\left|\sum_{i=0}^{n-1} \exp(-j\omega_d\tau_i)u_s(\tau_i + \lambda)\right|^2 d\lambda \underset{H_0}{\overset{H_1}{\gtrless}} \gamma. \tag{20}$$

We have compiled three optimal detector structures for range spread targets. The actual detector structure which is most applicable to UWB depends on the particular targets of interest to the radar designer and feasibility of implementation.

5. CONCLUSIONS

The purpose of this report is to examine general aspects of the detection of UWB signals and cite areas for further study. We determined detectors assuming white Gaussian noise and correlated noise, found a detection structure which generalizes a conventional Doppler filter bank, and illustrated a method to compute the GMTI coefficients. We also examined the effects of increasing bandwidth on clutter resolution and compared optimal and suboptimal detectors for single pulse echos. We found that the optimal processor can result in substantial improvement in subclutter visibility of the target echo. We also examined range spread or distributed targets. Cases of completely known and incomplete statistical knowledge of the target profile were treated.

The determination of the parameters of many of the detection structures discussed, such as optimal weights of a filter bank, has been shown to depend on the statistical characterization of the signal return which, in turn, depends on many factors. Signal characteristics (such as form and bandwidth), target characteristics (such as form and velocity), and the environment (which, along with various radar parameters, determines the degree of clutter and spatial correlation and the signal propagation paths) are all important factors among others that need to be further studied to properly design an UWB radar. Propagation models of UWB signals should be developed and their validity tested against actual data. These models can then be used to determine the parameters necessary to develop viable detectors. These parameters can be determined via appropriate models and compared by using actual data to assess their usefulness.

The finding in both Ref. 3 and in this paper that UWB radars yield a processing gain over conventional radars for targets which have numerous scatterers may imply that UWB radars have unique advantages.

It is recommended that future emphasis be placed on the phenomenological aspects of UWB radar signals. Models should be developed which accurately reflect the propagation of signals and the effects of clutter so that detectors can be properly designed.

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